An Analysis of the *Maryland PreK-8 Voluntary State Curriculum in Mathematics*

A Report submitted by Achieve to the Maryland Department of Education on May 28, 2009

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ACHIEVE'S STANDARDS REVIEW METHODOLOGY

Achieve has been conducting reviews of standards for ten years by benchmarking a state's Academic Standards to "exemplary standards." For purposes of this review, Achieve's *American Diploma Project (ADP) K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics*, the National Mathematic Advisory Panel *Foundations for Success* (NMAP), the National Assessment of Education Progress (NAEP) *Mathematics Framework* 2009—Grades 4 and 8—and the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* are the standards that were used as the basis of comparison for the review of Maryland's *PreK- 8 Voluntary State Curriculum (VSC) in Mathematics*.

THE DEVELOPMENT OF THE AMERICAN DIPLOMA PROJECT (ADP) K-8 BACKMAPPED STANDARDS IN MATHEMATICS

The American Diploma Project commissioned leading economists to examine labor market projections for the most promising jobs—those that pay enough to support a small family and provide real potential for career advancement—to pinpoint the academic knowledge and skills required for success in those occupations. ADP then surveyed officials from 22 occupations, ranging from manufacturing to financial services, about the skills they believe are most useful for their employees to bring to the job. Following those conversations, ADP worked closely with two- and four-year postsecondary leaders in the partner states to determine the prerequisite English and mathematics knowledge and skills required for success in entry-level, credit-bearing courses in English, mathematics, the sciences, and the humanities. The resulting ADP Benchmarks reflect an unprecedented convergence in what these employers and postsecondary faculty say are needed for new employees and freshmen entering credit-bearing coursework to be successful. Following the creation of the ADP Benchmarks, in 2008 Achieve published on its Web site K-6 standards and model middle school two-year course standards that were back-mapped from and vertically aligned with the ADP Benchmarks to ensure students are prepared to take on the ADP Benchmarks when they enter high school[1].

THE CRITERIA USED FOR THE EVALUATION OF MARYLAND'S VOLUNTARY STATE CURRICULUM IN MATHEMATICS

The development of the ADP Benchmarks, as well as lessons learned over ten years of analyzing state standards; provide Achieve content experts with a comprehensive view of the important characteristics shared by high quality standards. The resulting criteria, which are used in all Quality Reviews and in this evaluation of Maryland's VSC in Mathematics, include **rigor, coherence, focus, specificity, clarity/accessibility, and measurability.**

[1] The ADP Benchmarks in mathematics are currently being updated. After they are published in the fall of 2009, the Backmapped K-6 and Middle School Course Standards will be updated.

The Criteria Used for the Evaluation of Maryland's Voluntary State Curriculum in Mathematics

CRITERIA	DESCRIPTION
Rigor—What is the intellectual demand of the standards?	Rigor is the quintessential hallmark of exemplary standards. It is the measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in credit-bearing college courses without remediation and in entry-level, quality high-growth jobs. For Achieve's purposes, the ADP Benchmarks represent the appropriate threshold of rigor.
Coherence—Do the standards convey a unified vision of the discipline, do they establish connections among the major areas of study, and do they show a meaningful progression of content across the grades?	The way in which a state's College and Career Ready Standards are categorized and broken out into supporting strands should reflect a coherent structure of the discipline and/or reveal significant relationships among the strands and how the study of one complements the study of another**.** If College and Career Ready Standards suggest a progression, that progression should be meaningful and appropriate across the grades or grade spans.
Focus—Have choices been made about what is most important for students to learn, and is at the amount of content manageable?	High quality standards establish priorities about the concepts and skills that should be acquired by graduation from high school. Choices should be based on the knowledge and skills essential for students to succeed in postsecondary education and the world of work. For example, in mathematics choices should exhibit an appropriate balance of conceptual understanding, procedural knowledge and problem solving skills, with an emphasis on application, and in English standards should reflect an appropriate balance between literature and other important areas such as informational text, oral communication, logic, and research. A sharpened focus also helps ensure that the cumulative knowledge and skills students are expected to learn is manageable.
Specificity—Are the standards specific enough to convey the level of performance expected of students?	Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive. Standards that maintain a relatively consistent level of precision ("grain size") are easier to understand and use. Those that are overly broad or vague leave too much open to interpretation, increasing the likelihood that students will be held to different levels of performance, while atomistic standards encourage a checklist approach to teaching and learning that undermines students' overall understanding of the discipline. Also, standards that contain multiple expectations may be hard to translate into specific performances.
Clarity/Accessibility—Are the standards clearly written and presented in an error free, legible, easy-to-use format that is accessible to the general public?	Clarity requires more than just plain and jargon-free prose, which is free of errors. Standards must be communicated in language that can gain widespread acceptance not only by teachers and other educators but also by employers, parents, school boards, legislators, and others who have a stake in schooling. A straightforward, functional format facilitates user access.

CRITERIA	DESCRIPTION
Measurability—Is each standard measurable, observable, or verifiable in some way?	In general, standards should focus on the results, rather than the processes of teaching and learning. The College and Career Ready Standards should make use of performance verbs that call for students to demonstrate knowledge and skills and should avoid using those that refer to learning activities, such as examine, investigate, and explore, or to cognitive processes, such as appreciate.

MAJOR FINDINGS & RECOMMENDATIONS

OVERVIEW

To evaluate the **rigor, coherence, focus, specificity, clarity/accessibility, and measurability** of Maryland's *PreK-8 Voluntary State Curriculum (VSC) in Mathematics*, Achieve reviewers considered the most recent version sent to Achieve in October 2008. Achieve reviewers compared this version with the Achieve *American Diploma Project (ADP) K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics*, the National Mathematic Advisory Panel *Foundations for Success* (NMAP), the National Assessment of Education Progress (NAEP) *Mathematics Framework* 2009—Grades 4 and 8—and the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*.

Maryland's *PreK-8 VSC in Mathematics* is organized into multiple layers. It is first organized into seven Standards: Knowledge of Algebra, Patterns, and Functions; Knowledge of Geometry; Knowledge of Measurement; Knowledge of Statistics; Knowledge of Probability; Knowledge of Number Relationships and Computation/Arithmetic; and Processes of Mathematics. The seven Standards are then followed by Topics, Indicators, and Objectives. Some Objectives are further described by Assessment Limits.

Since the Objectives and associated Assessment Limits provide the greatest level of detail, these were the primary focus for this review. Achieve was not asked to include in its analysis associated sample problems embedded in the PreK-8 VSC, which exist on Maryland's Web site.

Achieve's major findings from this review are as follows:

- **Maryland's *PreK-8 VSC in Mathematics* is well aligned with the essential procedural content of the *ADP K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics*.**

  Throughout the major mathematical strands, the *PreK-8 VSC* effectively addresses the procedural skills in mathematics. Most backmapped ADP Benchmarks addressing procedural skills have well aligned counterparts in the *PreK-8 VSC*. For example, in the Grade 5 ADP Number strand, students are expected to "Add fractions with unequal denominators by rewriting them as equivalent fractions with equal denominators." In the *PreK-8 VSC*, students are expected to "Add and subtract proper fractions and mixed numbers with answers in simplest form." Furthermore, through the use of assessment limits, the *PreK-8 VSC* clearly communicates proficiency levels for given Objectives.

- **Attention to focus and coherence will further improve Maryland's *PreK-8 VSC in Mathematics*.**

  In its revision process, Maryland is encouraged to create greater balance in the focus of its document by more clearly addressing the conceptual underpinnings of the content and by increasing the requirement for students to apply their learning by solving complex problems.

To do so, Maryland is encouraged to integrate the Processes of Mathematics Strand into the content strands in a manner that balances application, conceptual understanding (which includes reasoning and connections), and procedural fluency. Because the expectations in this strand are not currently connected to the content strands, these processes may be devalued or over valued in classroom instruction. By integrating Standard 7 with the content strands, Maryland provides districts and teachers with clear guidance as to their relative importance. Furthermore, while they were not considered for this review, Maryland provides sample problems for some Objectives on its Web site. Where these exist, the expectations for conceptual understanding and application of content are clearer. Maryland is encouraged to expand their use, thereby clarifying expectation levels and thus the accessibility of the document.

Regarding coherence, the nature of the *PreK-8 VSC* is to "spiral" or repeat the learning of skills and concepts with a nominal increase in difficulty from year to year. This is especially noticeable in the Number strand. To make the *PreK-8 VSC* more coherent, Maryland is strongly encouraged to set mastery levels for student learning by bringing closure to content in a reasonable timeframe rather than repeating content year after year—ensuring a smooth and logical progression of knowledge and skills across the grades.

- **Overall, there is good alignment between the *PreK-8 VSC* and the ADP Backmapped Benchmarks, the NCTM Focal Points, and the NAEP Objectives in the area of Probability and Statistics, particularly in the earlier grade spans.**

Maryland is to be commended for attending to this content strand in such a thorough manner. This content is not addressed as completely in many states as Maryland has in the *PreK-8 VSC*. For example, in 1.4.A.1, Maryland requires students to "Organize and display data to make line plots using a variety of intervals," is often not addressed until middle school in many states.

FINDINGS AND RECOMMENDATIONS BY CRITERIA

Criterion 1—RIGOR: What is the intellectual demand of Maryland's *PreK-8 VSC in Mathematics*?

Rigor is the quintessential hallmark of exemplary standards; it is a measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in the next level of mathematics. For the purposes of this report, four national exemplars represent the appropriate threshold of rigor: the *ADP K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics*, the National Assessment of Education Progress (NAEP) *Mathematics Framework* 2009, the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, and the recommendations from the National Mathematics Advisory Panel *Foundations for Success* (NMAP).

❖ ***The Maryland PreK-8 VSC in Mathematics generally defines rigorous expectations around the procedures of mathematics. There are a few gaps that remain between Maryland's Objectives and standards defined in the national exemplars.***

With respect to conceptual understanding and the application of content, Maryland's Objectives are somewhat less rigorous than the national exemplars in all five of the major exemplar strands addressed: Number, Measurement, Algebra, Geometry, and Probability and Statistics. Specific comments on Maryland's Standards follow, with specific commentary organized by ADP strand and exemplar (grade bands K-3, 4-5, and 6-8 are aggregated within the strands):

- **Number**

ADP – An issue with respect to rigor exists in comparing the *PreK-8 VSC* to the ADP K-8 Benchmarks in Number. Since the *PreK-8 VSC* Objectives for Number are primarily skill based, they do not sufficiently address the conceptual understanding associated with number. For example, the *PreK-8 VSC* does not explicitly address the concept of a rational number, the existence of irrational numbers, and properties of numbers and operations—including the properties of addition, additive and multiplicative identities, the associative property of multiplication, properties of zero, and inverse properties, which are fundamentally critical to the later development of algebraic thinking. Although estimation content resides within its own strand under the VSC Standard "Knowledge of Number Relationships and Computation/Arithmetic," rounding and mental calculations are not explicitly addressed, especially when using 10, 100, 1000 in computations. The use of technology is addressed in the description of Standard 6 but not in the specific context of checking for accuracy, which is a critical learning for students. The conceptual generalized model of fractions as lengths on the number line is not expressed. Finally, algorithms are expected to be applied, but there is no evidence that students are expected to explain why they work. Without such an Objective, students may learn the algorithm for its own sake rather than for the sake of intentional application, thus reducing the rigor of the Objective.

NMAP – Overall, the *PreK-8 VSC* addresses the content expected in the NMAP benchmarks. In particular, the *PreK-8 VSC* effectively addresses the efficient use of the addition and

subtraction of whole numbers and fractions. In addition to noting the NMAP benchmarks which do not align well with the *PreK-8 VSC*, Maryland is encouraged to look closely at its Objectives around the comparison of different representations of fractions, decimals, and percents as outlined in NMP.F.1 and NMP.F.2. Currently, the *PreK-8 VSC* does not expect the same level of fluency or conceptual understanding in this area, which is foundationally critical for later development of algebraic thinking.

NAEP – Across Grades K-8, the VSC aligns well with the NAEP objectives in the area of Number Sense. By the end of Grade 4, the *PreK-8 VSC* meets the NAEP objectives in the areas of Number Sense, Numerical Operations, and Properties of Numbers and Operations. In Grades 5-8, the *PreK-8 VSC* aligns with the NAEP objectives in the areas of Number Sense and Ratio and Proportional Reasoning.

Generally, alignment in the area of Estimation is restricted to procedure. Maryland students are expected to estimate the solutions to addition, subtraction, multiplication, and division problems, but they are not expected to know when to estimate or to master common strategies, such as front end rounding. NAEP objectives in the area of Ratio and Proportional Reasoning have few if any counterparts in the *PreK-8 VSC* through the end of Grade 4 only. The same is true in Grades 5-8 in Number Operations and Properties of Numbers and Operations.

Overall, the *PreK-8 VSC* is more focused on the type of numbers used, while NAEP is more focused on what is done with the numbers. For example, the VSC has multiple Objectives for putting different types of numbers on the number line (4.1.C; 5.1.C; 6.1.C ...), while NAEP has standards about representing numbers, decomposing numbers, and connecting various representations of numbers. In its revision process, Maryland is encouraged to focus on Estimation and on how numbers are used.

NCTM – The *PreK-8 VSC* aligns better with the NCTM Focal Points in the earlier grades than it does in later grades. More specifically, the *PreK-8 VSC* has generally good alignment with NCTM's "Number and Operation" and "Number, Operation, and Algebra" strands in Grades K-3. In Grades 4 and 5, alignment continues to be strong in Number and Operations. In several cases, the VSC goes beyond the Focal Points when the focus is on procedural skill.

As was mentioned earlier in this document, the primary departure between the *PreK-8 VSC* and NCTM is in the focus on procedural skill rather than a balanced approach with respect to conceptual understanding. The Focal Points provide for a deeper study of their properties, with a focus on understanding and application. The *PreK-8 VSC* focuses primarily on skill development, as seen in the comparison with Focal Point NO.6-1. NCTM is concerned with understanding the meaning of the numbers and the procedures to solve problems, where the *PreK-8 VSC* limits its focus to skills.

- **Measurement**

ADP – Overall, the *PreK-8 VSC* Objectives through Grade 6 could be more rigorous. While they include an Objective that addresses selecting and using appropriate units, the rigor and

conceptual underpinnings of the backmapped ADP Benchmarks are unclear. For example, where ADP MS1.C1.a calls for students to "Recognize that measurements of physical quantities must include the unit of measurement, that most measurements permit a variety of appropriate units, and that the numerical value of a measurement depends on the choice of unit; apply these ideas when making measurements," the *PreK-8 VSC* requires students to "select and use appropriate tools and units." While it is possible that Maryland's assessments expect students to justify their choices of measurement units, and therefore demonstrate a "recognition that most measurements permit a variety of appropriate units and that the numerical value of a measurement depends on the choice of unit," it is not clear from the Objective alone. In addition, the *PreK-8 VSC* does not address the need for students to understand the relationships among one-, two-, and three-dimensional units of measurement. Precision and accuracy are not included in the context of measurement in the *PreK-8 VSC*. Computation using lengths, weights, capacities, and time are not specifically addressed in the *PreK-8 VSC* measurement Standard at any grade level. Computation with a variety of units is important to the development of future algebraic skills, thus the absence of such Objectives should be addressed.

NMAP – With respect to NMAP, differences in the anticipated grade levels at which the skill should be mastered exist. The *PreK-8 VSC* expects students to master similar content but at a later grade than prescribed by NMAP. For example, in NMP.GM.1, students are expected to solve measurement/geometric problems based on triangles and "all quadrilaterals having at least one pair of parallel sides," by the end of Grade 5. Prior to Grade 5, the only shape the *PreK-8 VSC* requires students to analyze is the rectangle.

NAEP – Two primary themes emerge in the NAEP comparison. First, the two documents align well with respect to the skills of measurement. Second, the alignment is less strong when considering the broader ideas or topics, particularly with respect to number systems. For example, NAEP M.4.1.b asks students to "Compare objects with respect to a given attribute, such as length, area, volume, time, or temperature," while the *PreK*-8 *VSC* has multiple objectives like, "1.1.B.1.b) Identify and compare units of capacity using cups and gallons; 1.1.B.1.c) Compare and order objects by weight in pounds using a spring scale and a bathroom scale; and 2.1.A.1.e) Identify and compare the weight of objects to the nearest pound." The *PreK-8 VSC* approach could lead to a reductive curriculum that is overly skill based at the expense of conceptual understanding.

NCTM – The *PreK-8 VSC* provides a much more specific and clearer set of standards around Measurement than does the NCTM. Overall, there is good alignment between the two, with a number of *PreK-8 VSC* standards lacking clear counterparts in the NCTM due to their increased specificity.

- **Algebra**

ADP – While most of the *PreK-8 VSC* Objectives in the Algebra strand have counterparts in the backmapped *ADP K-6 and Two-Year Middle School Backmapped Benchmarks*, there is a concern about rigor. The applications of algebra in the *PreK-8 VSC* tend to be at a more

skill-based level, where the application of algebra in the backmapped ADP Benchmarks tends to require a higher level of cognitive demand by the student. For example, in ADP MS2.D3.a, the student must "Recognize that in the form $f(x) = mx + b$, m is the slope, or constant rate of change of the graph of f, that b is the y-intercept and that in many applications of linear functions, b defines the initial state of a situation; express a function in this form when this information is given or needed." Yet, the VSC Objective with the best alignment is 8.1.C.2, which requires students to "Analyze linear relationships: b) Determine the slope of a linear relationship represented numerically or algebraically." The most significant differences between these requirements lies in what students are expected to do with linear relationships. Where the backmapped ADP Benchmark expects students to know how to write a linear function in a specified form when given certain information and to know what the information means, the VSC Objective simply asks students to determine a slope computationally when given numbers or a graph. In other words, where the backmapped ADP Benchmark expects students to take given information, analyze it for its linear properties, and then represent it symbolically as a linear function, the VSC Objective asks students to determine a slope.

The *PreK-8 VSC*'s treatment of the conceptual understandings of algebra needs to be addressed. For example, the *PreK-8 VSC* has no counterpart for ADP MS2.C2.a, which requires students to "Analyze expressions to identify when an expression is the sum of two or more simpler expressions (called terms) or the product of two or more simpler expressions (called factors). Analyze the structure of an algebraic expression and identify the resulting characteristics."

NMAP – The National Mathematics Advisory Panel's *Benchmarks for the Critical Foundations of Algebra* addresses the foundations of Algebra in Number and Geometry. Commentary on the *PreK-8 VSC's* alignment to NMAP can be found in these two sections of this report.

NAEP – Most NAEP objectives have counterparts in the *PreK-8 VSC*, which leads to an overall rating of good alignment. Furthermore, the same issue with respect to cognitive demand raised earlier in this report emerges among those NAEP objectives which have little or no alignment to the *PreK-8 VSC*. For example, where the NAEP objective A.8.4.c requires students to "Analyze situations or solve problems using linear equations and inequalities with rational coefficients symbolically or graphically," the associated VSC standards require students to "Use ratios and unit rates to solve problems," which lacks the application of linear relationships to a contextualized problem.

NCTM – While there are counterparts in the *PreK-8 VSC* to NCTM, thus suggesting generally good alignment, differences exist about the level of cognitive demand required of students by the *PreK-8 VSC* when compared to NCTM. Focal Point A8 provides a good example. The *PreK-8 VSC* has the foundational content that leads to mastery of this Focal Point, but generally the *PreK-8 VSC* content is a precursor to the Focal Point content. More specifically, the VSC references function tables with two-operation rules, while Focal Points discusses special cases of linear equations ($y/x = k$). The first is a very concrete

representation of a linear relationship, while the second is an abstract representation of a specific type of linear relationship, requiring a deeper understanding.

- **Geometry**

ADP – Most of the *PreK-8 VSC* have counterparts in the backmapped ADP Benchmarks in Geometry; therefore, there exists good alignment overall. Those backmapped ADP Benchmarks with little or no alignment have a higher level of cognitive demand than is found in the *PreK-8 VSC*. Cognitive demand is a concern in two areas of geometry—representations of three-dimensional objects in two-dimensional space and in general geometric transformations. Neither area is well addressed in the *PreK-8 VSC*. In cases where they are addressed, the level of cognitive demand is not as high as in the backmapped ADP Benchmarks. For example, where ADP G.5.3.a states that students will be able to, "Represent and work with rectangular prisms by means of orthogonal views, projective views, and nets," the *PreK-8 VSC* counterparts restrain the requirement to, "Compare a plane figure to surfaces of solid geometric figures," (VSC 4.2.B.2.) In these cases, Maryland is encouraged to focus its revision on increasing the cognitive demand of its standards by including greater specificity as to the types of comparisons students should make.

NMAP – The National Mathematics Advisory Panel's *Benchmarks for the Critical Foundations of Algebra* combines Geometry and Measurement. Please see the remarks under Measurement for information on Geometry.

NAEP – Most of the NAEP objectives have counterparts in the *PreK-8 VSC*, which leads to a rating of good alignment. However, of the 17 NAEP objectives with little or no alignment to the *PreK-8 VSC*, the issue of cognitive demand emerges once again. Where the NAEP objectives expect a higher level of cognitive demand, the *PreK-8 VSC* tends to focus on foundational skills. For example, in NAEP G.8.2.d, students are expected to "predict the results of combining, subdividing, and changing shapes of plane figures and solids." The *PreK-8 VSC* counterpart restricts the student requirement to "Estimate and determine area of a composite figure."

NCTM – Overall, there is strong alignment to NCTM with one exception. All Focal Points had good or excellent alignment with the *PreK-8 VSC* except for Focal Point GMA.5. With respect to this comparison, *PreK-8 VSC* Objectives from Grades 4-8 were needed to create an alignment with this Grades 4-5 Focal Point. Because middle school VSC Objectives were found to align with a Grades 4-5 Focal Point, there is a question as to the level of cognitive demand in these VSC Objectives.

- **Probability and Statistics**

ADP – Overall, there is good alignment between the *PreK-8 VSC* and the backmapped ADP Benchmarks in the area of Probability and Statistics, particularly in the earlier grade spans. As the grade level increases, the strength of alignment decreases slightly. For example, while the *PreK-8 VSC* expects students to display data using frequency tables in Grade 6, it is not explicit about the requirement for students to understand or compute relative and

cumulative frequencies. The *PreK-8 VSC* expects students to interpret, organize, and display data using box and whisker plots but does not explicitly require students to find and interpret the median, upper quartile, lower quartile, and interquartile range of a set of data. In order to construct a box and whisker plot, all of these measures must be found. Without explicitly requiring this of students, the question as to how they will create the box and whisker plots emerges. As it is possible to create them with the use of technology, Maryland is encouraged to be more specific as to how the plots should be created.

NMAP – The National Mathematics Advisory Panel's *Benchmarks for the Critical Foundations of Algebra*, to which the *PreK-8 VSC* was compared, does not include a Probability and Statistics strand.

NAEP – Overall, there is strong alignment between NAEP and the *PreK-8 VSC* in this area.

NCTM – All Focal Points had good or excellent alignment with *PreK-8 VSC* counterparts.

Recommendations for improvement:

- Maryland should increase the cognitive demand of its Objectives by focusing on conceptual understanding and problem solving either through a change in Objectives or through an increased use of sample problems that address these skills.

Criterion 2—COHERENCE: Do the content standards convey a unified vision of the discipline, do they establish connections among the major areas of study, and do they show a meaningful progression of content across the grades?

The way in which a set of standards is categorized and broken out into supporting strands should reflect a coherent structure of the discipline and/or reveal significant relationships among the strands and how the study of one complements the study of another. The *PreK-8* VSC should suggest a progression of content that prepares a student for high school.

❖ ***Maryland's PreK-8 VSC generally conveys a unified vision of mathematics and establishes connections among the major areas of study.***

Maryland clearly and explicitly addresses procedural content in the areas of Number, Measurement, Algebra, Geometry, and Probability and Statistics. Less explicit are the expectations that address the conceptual underpinning of each Objective. In Number, as noted earlier, the *PreK-8 VSC* does not explicitly address the concept of a rational number, the existence of irrational numbers, and properties of numbers and operations—including the properties of addition, additive and multiplicative identities, the associative property of multiplication, properties of zero, and inverse properties. However, Maryland has created a set of standards that evidences some classic areas of integration or connection among the mathematical strands. These connections include counting and pattern recognition (number-algebra) and analytic geometry. All in all, the *PreK-8 VSC* does a strong job of presenting a unified vision of mathematics and establishing connections among the major areas of study.

❖ ***In some areas, Maryland's PreK-8 VSC repeats Objectives across grade levels with only minor changes rather than defining and developing a progression of content and skills across PreK-8.***

The organizational structure of the Maryland standards suggests a repetitive structure for conveying the progression of content and skills in mathematics. While grade level based proficiency is apparent in the assessment limits, there is no clear communication of when students are expected to have mastered particular content. Rather, topics are continued from one grade to the next without sufficient closure. The Plane Geometry strand in Standard 2 provides an illustration of repetition rather than significant progression. For example, VSC 3.2.A.1.a, 4.2.A.1.a, 5.2.A.1.a, and 6.2.A.1.a carry on the identification and description of various lines and line segments across four years.

Within VSC Standard 1—Knowledge of Algebra, Patterns, and Functions—there are only a few Objectives which repeat where the backmapped ADP Benchmarks do not. The following VSC Objectives repeat and thus have no counterparts in the backmapped ADP Benchmarks, as the backmapped ADP Benchmarks do not continue benchmarks focused on patterns after Grade 3. Beginning in Grade 4, students must represent and analyze numeric patterns of 3, 4, 6, 7, 8, or 9 starting with any whole number 0-100 using skip counting. Students must also create one-operation (+ or –) function table to solve a real world problem, complete a function table using a one-operation (+,–, x, / with no remainders) rule for whole numbers 0-50, and describe the relationship that generates a one-operation rule. Also in Grade 4, students must generate a rule for the next level of the growing pattern using at least three levels but no more than five levels, generate a rule for a repeating pattern using no more than four objectives in the core of the

pattern, and create a non-numeric growing or repeating pattern. In Grade 5, students must create a one-operation (x, / with no remainders) function table to solve a real world problem, complete a one-operation function table using whole numbers with +, –, x, / (with no remainders) or use decimals with no more than two decimals places with +, – for numbers 0-200. Finally, they must apply a given two-operation rule for a pattern using two operations (+, –, x) and whole numbers 0-100.

With respect to Probability, the *PreK-8 VSC* introduces content earlier than the backmapped ADP Benchmarks. Where ADP does not address probability in Grades K-3, the VSC does, with Objectives 1.5.A.1.a, 2.5.A.1.a, 3.5.A.1.a, 3.5.A.1.b, 3.5.B.1.a. Some *PreK-8 VSC* Objectives from Grades K-5 align with backmapped ADP Benchmarks for middle school. They are 5.5.A.1.a, 4.5.B.1.a, and 5.5.B.1.a 7. The inclusion of these topics at earlier grades does not increase the rigor of the *PreK-8 VSC* but does suggest a slightly different order of when students are first introduced to these topics.

Recommendations for improvement:

- Maryland should consider including in the *PreK-8 VSC* the knowledge and skills related to Number that have been identified in this report to create a more unified vision of mathematics.
- A progression of knowledge and skills should be defined and developed coherently and brought to closure as early as possible across Grades PreK-8, rather than repeating Objectives, with only minor variation, from one grade level to the next.

Criterion 3—FOCUS: Have choices been made about what is most important for students to learn, and is the amount of content manageable?

High quality standards establish priorities about the concepts and skills that should be acquired before entering high school. Choices should be based on the knowledge and skills essential for students to advance to the next level of understanding. In mathematics, choices should exhibit an appropriate balance of conceptual understanding, procedural knowledge, and problem solving skills—with an emphasis on application. A sharpened focus also helps ensure that the cumulative knowledge and skills students are expected to learn is manageable.

❖ ***Maryland's PreK-8 VSC is generally manageable.***

The total body of expectations would most likely be addressed in an instructional program spanning Grades PreK-8. Some students will be able to complete these standards in earlier grades and should have accelerated options for the final years before high school mathematics study or be allowed to begin study of high school mathematics early. With this set of standards mastered, a student would be prepared for the procedural skills of an Algebra I course.

❖ ***Maryland's PreK-8 VSC could more effectively balance conceptual understanding, procedural fluency, and problem solving.***

As noted previously in this analysis, there is a significant gap around the issue of balance. Where the backmapped ADP Benchmarks require students to learn conceptual understandings, procedural skills, and to apply their learning, the *PreK-8 VSC's* focus lies almost entirely in the realm of procedural skill. In each of the content strands, Objectives are consistently lacking which require students to understand underlying concepts and apply knowledge. In the Communication strand of the process standards, students are expected to "use multiple representations to express mathematical concepts using concrete, pictorial, tabular, graphical, and algebraic methods." This standard could be the vehicle for expressing the importance of conceptual understanding were it more directly connected with the content standards. VSC Standard 7—Processes of Mathematics—provides a focus on reasoning and problem solving. Yet, without integration into the content standards, teachers and districts may over- or under-emphasize them.

Recommendations for improvement:

- Maryland may want to consider whether it wants to broaden its Grades PreK-8 Objectives to meet the needs of students who progress more quickly through the content or allow students to begin studying high school content prior to high school.
- Maryland should integrate its Standard 7 Objectives within Standards 1-6 to create a better balance among conceptual understanding, procedural skills, and problem solving. While the existence of sample problems on its Web site increases the focus of the PreK-8 VSC, Maryland should either significantly increase their number or change the language of the Objectives to incorporate Standard 7.

Criterion 4—SPECIFICITY: Are the Content Expectations specific enough to convey the level of performance expected of students?

Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive. Standards that maintain a relatively consistent level of precision ("grain size") are easier to understand and use. Those that are overly broad or vague leave too much open to interpretation, increasing the likelihood that students will be held to different levels of performance, while atomistic standards encourage a checklist approach to teaching and learning that undermines students' overall understanding of the discipline. Also, standards that contain multiple expectations may be hard to translate into specific performances.

❖ ***The proposed Maryland PreK-8 VSC in Mathematics includes a number of Objectives that are atomistic in nature.***

When taken collectively, there are a number of instances where the Maryland Objectives are atomistic in nature. This seems to be the result of the document's attempt to repeat one concept across multiple grade levels with only small changes. For example, in Grades PreK-3, students are expected to perform the same skills with only slight variation from one year to the next.

In VSC Standard 6—Knowledge of Number Relationships and Computation/Arithmetic—the *PreK-8 VSC* has more Objectives than the backmapped ADP Benchmarks. In Grade 1, students must estimate quantities up to 50 using the term "about." They must also develop strategies for addition and subtraction basic facts such as: counting on, counting back, making 10, doubles, and doubles plus one. In Grade 2, students must estimate quantities up to 100 using a reference point such as 10 and the terminology "about." They must also demonstrate proficiency with addition and subtraction basic facts using a variety of strategies. In Grade 3, students must identify and use properties of multiplication including commutative, identity, or zero and whole numbers for numbers 0-20. In Grade 4, students must compare, order, and describe decimals with or without using the symbols (<, >, or =) for no more than three decimals with no more than two decimals places and numbers 0-100. In Grade 5, students must compare, order, and describe decimals with or without using the symbols (<, >, or =) for no more than four decimals with no more than three decimals places and numbers 0-100.

❖ ***Maryland's PreK-8 VSC is generally precise and specific, and the level of detail is generally uniform.***

Maryland's standards documents have multiple levels of specificity. As expected, the level of specificity increases as one moves from the Standards to the Assessment Limits. While the detail provided in the Assessment Limits is quite specific and includes parameters for the development of assessment items, instructional guidance could be helpful in further indicating where teachers might more fully address application and conceptual understanding that is not captured in the Assessment Limits.

Recommendation for improvement:

- Maryland should consider moving away from overly detailed or atomistic Objectives. For example, in Grade 5, students must compare, order and describe decimals with or without using the symbols (<, >, or =) for no more than four decimals with no more than

three decimals places and numbers 0-100. Rather, Maryland may want to write this Objective as, "Students will compare, order and describe decimals with or without using the symbols (<, >, or =)."

- As much as possible, guidance should be provided to teachers to meet the requirements of the Assessment Limits and to address application and conceptual understanding that in some cases, go beyond the Assessment Limits.

Criterion 5—CLARITY/ACCESSIBILITY: Are the standards clearly written and presented in an error-free, legible, and easy-to-use format that is accessible to the general public?

Clarity requires more than just plain and jargon-free prose, which is free of errors. Standards must be communicated in language that can gain widespread acceptance not only by teachers and other educators but also by employers, parents, school boards, legislators, and others who have a stake in schooling. A straightforward, functional format facilitates user access.

❖ ***The language and format of Maryland's PreK-8 VSC are generally clear and use language appropriate for any school-related audience.***

The standards rarely use unnecessary jargon or highly technical language, and few unfamiliar terms. On the whole, the standards are clearly written, with a few exceptions are noted below:

- Overly specific standards, particularly those intended to define the assessment limits, may make the standards less understandable to the general public. For example, consider the assessment limit to 8.1.A.b that states: "Provide the nth term no more than 5 terms beyond the last given term using the recursive relationship of geometric sequences."
- For example, it is not clear how one should interpret the statement, "Apply given formulas to problem solving situation," accompanying 8.1.B.2.f. Does it mean a formula will be provided and a student must use it? Does it mean that the student will determine the proper formula to use from among a set of formula given a particular situation?

❖ ***Maryland's PreK-8 VSC needs to be more clear and accessible, particularly with regard to the expected level of cognitive demand.***

While reviewers found the layered organization of the VSC document difficult to navigate, it is Achieve's understanding that educators across the state find the organization user friendly. One exception to accessibility is that Objectives which do not have accompanying sample problems—which are found only on the State Board's Web site—do not clearly convey the expected level of cognitive demand.

Recommendations for improvement:

- Maryland should consider creating an alternative display for the Standards defined in the *PreK-8 VSC* documents. For example, the level of detail, particularly in the Assessment Limits, is critical for teachers to know how and at what level content will be assessed, but this information is in all likelihood not needed by the general public.
- Maryland should clearly communicate the expected level of cognitive demand associated with each Objective. Ancillary resources could be developed and/or enhanced, if desired, to define assessment limits and/or to provide examples that support the standards and objectives.

Criterion 6—MEASURABILITY: Is each standard measurable, observable, or verifiable in some way?

In general, standards should focus on the results, rather than the processes of teaching and learning. Like the *ADP K-8 Backmapped Benchmarks*, the *PreK-8 VSC* should target preparation for success in a ninth grade mathematics course. The *PreK-8 VSC* should make use of performance verbs that call for students to demonstrate knowledge and skills and should avoid using those that refer to learning activities, such as examine, investigate, and explore, or to cognitive processes, such as appreciate.

- ***Maryland's PreK-8 VSC are measurable or verifiable in some way.***

Except for the standards identified in this report that are not clear or need to be made more specific, each standard is measurable, observable, or verifiable in some way. In general, the Maryland standards focus on the results rather than the processes of teaching and learning. For example, performance verbs, such as "determine," "simplify," "evaluate," "represent," and "graph," are very common in the Algebra Objectives. Performance verbs, such as "describe," "analyze," "indentify," "classify," and "compare," are very common in the Geometry Objectives.

CONCLUSION

In September 2007, Achieve completed a Quality Review II of the *Maryland VSC – High School Mathematics*. In that analysis, Achieve found that the High School VSC aligned well with the ADP Benchmarks. In particular, with the inclusion of sample problems, which Maryland released to public, the level of cognitive demand was clarified. The state had responded to Achieve's recommendation in Quality Review I, that "the state work carefully to identify a range of exemplars that show the level of rigor of the expectations and the depth and breadth of content coverage intended by the standards." After an extensive revision process, Maryland successfully addressed this recommendation as found in the Quality Review II report, "Maryland has expanded and enhanced its VSC in a number of ways—by adding an Algebra II VSC; adding Additional Topics to its Algebra/Data Analysis, Geometry, and Algebra II VSC; including introductory text that clarifies not only format and purpose but also the importance of solving problems and reasoning processes as well as the use of technology; and developing a Web site that contains—among other things—publicly-released assessment items to help clarify the intended level of rigor of at least some of the VSC."

In comparing the *PreK-8 VSC* to the *ADP K-8 Backmapped Benchmarks*, Achieve found a discrepancy in the intended level of rigor and focus in the *PreK-8 VSC* with respect to cognitive demand and balance among conceptual understanding, procedural fluency, and solving problems. A similar concern existed following the Quality Review I of the High School VSC, which Maryland effectively addressed.

Maryland is now encouraged to address this concern in the *PreK-8 VSC*, as it partially addresses the essential content of the *ADP K-8 Backmapped Benchmarks for Mathematics* and does so particularly well in regards to the procedural skills of mathematics. By strengthening the alignment in the areas identified in this report, Maryland can make the necessary adjustments to ensure that the state has standards that will ensure that students achieve proficiency as exemplified in the backmapped ADP Benchmarks and in other nationally-recognized frameworks. Maryland is to be commended for continuing to revise its standards in an effort to create closer alignment with ADP, NAEP, NMAP, and NCTM, as alignment with these exemplars is certainly a worthy goal.

In revising the *PreK-8 VSC*, Maryland is encouraged to address six items to bring the VSC into strong alignment with nationally-recognized standards documents. Achieve recommends:

- increasing rigor by increasing the sophistication of mathematical content and cognitive demand;
- increasing coherence through consolidation of content;
- increasing focus through better balance among procedural skills, conceptual understanding, and solving problems;
- making the Objectives and associated Assessment Limits less overly specific; and
- making the document clearer and more accessible.

As a result of its thorough revision process, Maryland will ensure that its standards attend to the specific requirements of each exemplar by addressing these recommendations. In particular, in bringing its standards into even greater alignment with the backmapped ADP Benchmarks,

Maryland will ensure that its students are prepared for high school and ultimately graduate prepared for success in college and careers.

APPENDIX A: BIOGRAPHIES

The following Achieve staff and consultants in mathematics led the analysis and report development for Maryland.

ACHIEVE STAFF

KAYE FORGIONE, SENIOR ASSOCIATE, MATHEMATICS, ACHIEVE

Kaye Forgione joined Achieve as senior associate for mathematics in March 2001 where she leads Achieve's Standards and Benchmarking Initiatives involving mathematics. Prior to joining Achieve, Kaye served as assistant director of the Systemic Research Collaborative for Mathematics, Science and Technology Education (SYRCE), a project at the University of Texas at Austin funded by the National Science Foundation. Her responsibilities at the University of Texas also included management and design responsibilities for UTeach, a collaborative project of the College of Education and the College of Natural Sciences to train and support the next generation of mathematics and science teachers in Texas. Before her work at the University of Texas, Kaye was director of academic standards programs at the Council for Basic Education, a nonprofit education organization located in Washington, DC. Prior to joining the Council for Basic Education in 1997, Kaye worked in the K-12 arena in a variety of roles, including several leadership positions with the Delaware Department of Education. Kaye began her education career as a high school mathematics teacher. She taught mathematics at the secondary and college levels as part of adult continuing education programs. Kaye received a bachelor's degree in mathematics and education from the University of Delaware, a master's degree in systems management from the University of Southern California, and a doctorate in educational leadership from the University of Delaware.

JAMES MACDONALD, PROJECT ASSOCIATE, ACHIEVE

James Macdonald supports Achieve's Content & Policy Research work, editing and formatting documents, as well as by providing content support. Before joining Achieve in 2004, James was executive assistant and membership coordinator at the George C. Marshall Institute. At the same time, he served as an adjunct philosophy professor at Catholic University of America and Mount St. Mary's College. James also has taught philosophy courses at the University of Toledo while working on his graduate studies. He has had articles published in *Philosophical Writings UK* and the *Review of Metaphysics.* James received a bachelor's degree in philosophy and history from Ohio Northern University and a master's degree in philosophy from the University of Toledo. He pursued a doctorate in philosophy from Catholic University of America, where he has completed everything but his dissertation.

LAURA MCGIFFERT SLOVER, VICE PRESIDENT CONTENT & POLICY RESEARCH, ACHIEVE

Laura McGiffert Slover is Vice President of Content & Policy Research at Achieve, where she has senior responsibility for overseeing a number of Achieve's major initiatives. She supervises Achieve's Benchmarking Initiative, leads its work with states on building mathematics capacity, and oversees the organization's research agenda. Laura has extensive experience reviewing

academic standards and education policies in the United States and abroad, and she has written a number of reports and articles on the topic. Before joining Achieve in 1998, Laura was a high school English teacher in Eagle County, Colorado, where she was involved in the district's early efforts to develop standards and benchmark assessments. She also taught writing and composition at Colorado Mountain College. A native Washingtonian, Laura earned a bachelor's degree in English and American Literature from Harvard University; a master's in Education Curriculum and Instruction from the University of Colorado at Boulder; and a master's in Education Policy from Georgetown University. She currently serves as a member of the Board of Education of the District of Columbia.

DOUGLAS SOVDE, ASSOCIATE, MATHEMATICS, ACHIEVE

Douglas Sovde joined Achieve in 2008 as an associate for mathematics. His primary responsibilities include leading reviews of state mathematics standards in support of Achieve's Alignment Institutes and the American Diploma Project. Prior to joining Achieve, Doug spent 12 years in the Bellevue (WA) Public Schools as a teacher, an assistant principal and a principal. As a mathematics teacher, Doug taught courses from pre-algebra to AP Calculus BC. He later became an assistant principal at Bellevue High School and Sammamish High School, where he also supervised the mathematics departments. In 2006, Doug became the principal of Chinook Middle School, where he managed the development of a new curriculum in mathematics, science, and social studies. He also served as the liaison between the school district and the University of Washington's LIFE Center to provide staff development to principals and curriculum developers on instructional leadership, curriculum development and classroom instruction. Mr. Sovde earned a bachelor's degree in mathematics from the University of Washington, a master's degree in Curriculum and Instruction from Western Washington University, and his principal certification from the University of Washington's Danforth Educational Leadership and Policy Studies Program.

CHRISTINE TELL, SENIOR ASSOCIATE, ALIGNMENT, ACHIEVE

Christine Tell joined Achieve in 2004 as director of the American Diploma Project. Christine coordinates the activities of the 19 states engaged in the Alignment Institute Initiative. Prior to Achieve, Christine was Director of the Proficiency-based Admission Standards System (PASS) for the Chancellor's Office of the Oregon University System. PASS aligns students' proficiency in the K-12 system with the knowledge and skills needed for entry-level college success in Oregon's public universities. PASS has been nationally cited for pioneering methods in teacher judgment of student proficiency via collections of student work, and developing college admissions policies that include proficiency and state-level alignment of standards and assessment across K-12 and postsecondary. While in Oregon, Christine led a study to examine the effects of Oregon's statewide alignment: *The First Year: Student Performance on High School Benchmarks and Subsequent Performance in the First Year of College.* Before her work at the Oregon University System, she served as Director of the Oregon LEAD Project and developed the Goals 2000 plan for the state of Oregon. Christine's career in education includes over 15 years of teaching in elementary, middle, high school, and university classrooms. Christine received a bachelor's degree in American Studies from the University of Virginia, Mary Washington College, followed by graduate study at Exeter College, Oxford University in history and literature. She received her master's degree in special education and a doctorate in curriculum, instruction, and assessment from the University of Oregon.

CONTENT EXPERTS AND REVIEWERS IN MATHEMATICS

LINDA PLATTNER

Linda Plattner is the President and founder of Strategic Teaching, an organization that works with districts, foundations, and organizations to improve teaching and the systems in which they work. At Strategic Teaching, among many projects, Ms. Plattner currently helps the U.S. Department of Education by recruiting, training, and managing 120 presenters and the content of their presentations at the Department's Teacher-to-Teacher Summer Workshop and Training Corps. She has also worked in the Cleveland Public Schools for several years, facilitating their standards development, the alignment of those standards to Ohio's state Standards, drafting and implementing a public engagement effort around the standards, and facilitating the development of an online, standards-based report card built from the state standards. Prior to founding Strategic Thinking, Ms. Plattner was from 1997-2001 Director of Policy, Standards, and Instruction at the Council for Basic Education. She was also Mathematics Curriculum Coordinator, and prior to that a teacher, in the Moses Lake School District in the state of Washington. Ms. Plattner has a master's in humanities from California State University, as well as a master's in mathematics and a bachelor's in education from Central Washington University.

MARY LYNN RAITH

Mary Lynn Raith received her B.S in mathematics from Indiana University at Pittsburgh and her M.Ed. in mathematics education from the University of Pittsburgh. She is recently retired from the position of Mathematics Specialist in the Division of Instructional Support of the Pittsburgh Public Schools. As such, her responsibilities included leadership roles in curriculum development, textbook selection, design of alternative assessments, in-service program design and implementation, and coordination of mathematics programs across levels and schools. Ms. Raith was also the Co-Director of the Pittsburgh Reform in Mathematics Education project (PRIME), a K-12 professional development system. She has also been involved with a number of national projects, including the development of both the New Standards Reference Examination and the Portfolio project for the middle grades, the Assessment Communities of Teachers project (ACT), and the Alternative Assessment in Mathematics project (A^2IM). She has also worked extensively with both NCTM and NCEE on its America's Choice school design and has presented at numerous national conferences.